44 Shafts, journal and rolling bearings

44.1 Shafts and axles

44.1.1 Definitions

A *shaft* is a rotating member, usually of circular cross-section, used to transmit torque or motion. It provides the axis of rotation, or oscillation, of components such as gears, pulleys, cams, sprockets, cranks (*crankshafts* are not covered in this Unit), etc. and controls the geometry of their motion. According to the cross-section, a shaft can be of *solid* or *hollow section*; according to the shape, it can be *circular* or a *spline shaft*. An *axle* is a (in general, nonrotating) member which carries no torque and is used to support rotating components, such as wheels and pulleys; therefore it is *mainly subjected to bending* like it occurs for a rotating railway trolley. A *spindle* is a short shaft. Shafts (always rotating) transmit torque and can be subjected to bending, torsional and axial loads, as it occurs for a geared counter-shaft to be supported by two bearings.

The geometry of a shaft is generally that of a stepped cylinder (*Figure 44.1*). Bearings, gears, pulleys, and other members must always be accurately positioned: shaft shoulders are an excellent means of locating the shaft elements; for instance, they can be used to pre-

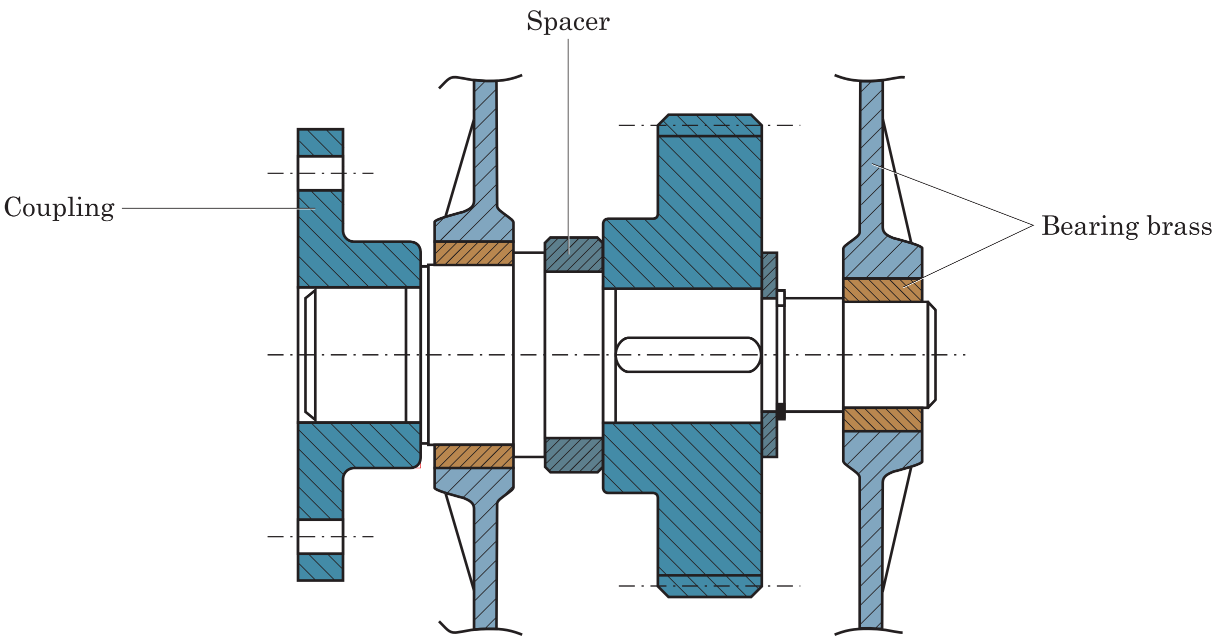


Fig. 44.1 - Some elements of a shaft. The shaft can transmit torque to another shaft by means of coupling.

load rolling bearings and to provide thrust reactions for the rotating elements. The design of slots for feathers or keys, holes for pins, and those shaft portions allowing to spline the hub of a rotating member must account for notch sensitivity. Some components, as for example cams, can be forged into the shaft surface; in such a case the shaft geometry has to be defined versus the tooling of these components.

About the shaft supported elements it would be known:

– the length of bearings;
– the length of pulleys, gears and other transmitting torque members;
– the distance between hubs and supports that has to be as short as possible, but still compatible with their operation and assembly, in order to minimize the shaft stresses.

In general, the shaft should be considered as a rectilinear elastic beam with two supports. Stresses due to forces or torques are referred to the barycentric axis of the shaft having a circular cross-section and, even in case of distributed loads, a concentrated force in the shaft section is assumed. The own weight of the shaft is considered only when its size is such to give rise to a significant contribution to the state of stress. In design, the geometry of the entire shaft is not needed because the stress analysis, at a specific point on a shaft, can be made using only the shaft geometry in the vicinity of that point. Once the critical areas are located, these are sized to meet the strengths requirement; then, the rest of the shaft is sized to meet the requirements of the shaft-supported elements.

44.1.2 Sizing

The strength of a material is its ability to withstand an applied load without failure. The material strength property in the design (**19-13**) or verification (**19-14**) of a shaft is usually referred to the allowable normal stress σ_{all}. [Note that the subscript "all" (allowable) is the translation of the Italian "amm" (*ammissibile*)]. When reference is made to the shear stress τ, as it occurs in the design or verification of a ductile material bar in torsion, the allowable shear stress to be considered versus the normal stress is (**19-15**) $\tau_{all} = \left(1/\sqrt{3}\right)\sigma_{all} = 0.577 \cdot \sigma_{all}$.

We have already seen in *Table 25.4* the mechanical properties of materials used in gearing. Mechanical properties of some typical structural materials are shown in *Table 44.1*. For shafts subjected to minor stresses, carbon steels having a tensile strength R_m (indicated also as ultimate stress σ_u) between 400 and 1000 MPa are used, while for those shafts subjected to high stresses, high strength alloy steel, capable of being hardened and tempered, having a tensile strength from 800 to 1400 MPa are used. The tensile strength range shown in *Table 44.1* is the probability that the specimen fails when the given level of stress is reached; if, for instance, a steel with a tensile strength $R_m = 650 \div 800$ MPa is considered, passing from the minimum value of 650 MPa, corresponding to a 5% failure probability, to higher values, the failure becomes more and more probable up to the maximum of 800 MPa, at which 95% failure probability exists for that specimen.

When the shaft is subjected to static loads, the allowable stress σ_{all} is the ultimate stress divided by (**19-12**) a factor of safety $n = 2.5 \div 5$; higher values of the factor of safety are used if the mechanical element is subjected to an impact load.

In case of variable loading (fatigue conditions), when neither the fatigue limit σ_f nor the fatigue strength on N cycles σ_N are known, σ_f can be expressed as a function of the ultimate tensile strength σ_u measured on a specimen; as a matter of fact, in the second volume of the text we quoted for steels:

$$\left.\begin{aligned} \sigma_f &= (0.4 \div 0.6)\sigma_u \qquad \text{<< steels – bending loads >>} \\ \sigma_f &= (0.3 \div 0.45)\sigma_u \qquad \text{<< steels – axial loads >>} \\ \sigma_f &= (0.23 \div 0.35)\sigma_u \qquad \text{<< steels – torsional loads >>} \end{aligned}\right\} \qquad \textbf{(19-18)}$$

where the lower values (0.4, 0.3, 0.23) have to be used for high-strength steels and the higher ones (0.6, 0.45, 0.35) for the low-strength steels. The above relationships point out that the fatigue limit decreases at first from alternating bending to tensile-compressive axial loads and then to alternating torsional loads. Apart from torsion which follows the relationship **19-15** between shear and axial stress, the fatigue limit of an axially loaded member is lower than the alternating bending fatigue because the extreme higher loaded fibers of the specimen (i.e. the shaft fibers which are farthest from the neutral axis) subjected to bending can discharge their stresses on the internal lower loaded fibers (close to the neutral axis). The allowable fatigue stress $\sigma_{all,f}$ is then determined by dividing the fatigue limit σ_f, obtained with **19-18** equations which are referred to the specimen, by the factor of safety $n = 2.5 \div 5$.

In *Paragraphs 19.11.2* and *19.11.3*, the actual fatigue strength was examined; this is the strength taking into account the actual operation of a member within the mechanical structure, i.e. under working conditions completely different from those measured on a smooth specimen of material. When the actual fatigue strength is known, the allowable fatigue strength is obtained by dividing the fatigue limit σ_f or the fatigue strength σ_N on N cycles by a lower factor of safety, i.e. $n = 1.5 \div 2$ (see *Example 19.9*).

Being in practice quite difficult to collect information needed to determine the actual fatigue, it is preferable to divide the ultimate tensile strength σ_u by a very high factor of safety (up to 9) to obtain the allowable fatigue strength $\sigma_{all,f}$; this value can be further increased to 11 when a significant contribution of the fatigue stress-concentration factor K_f is expected. K_f, that is the ratio of maximum stress in notched specimen to stress in notch-free specimen, is the most insidious factor among all those influencing the actual fatigue of a piece.

Shafts are usually subjected to bending-moments and torque (only in shafts while, in axles, the torque is zero), while shearing-forces and axial-forces are negligible. Diagrams of variation of these quantities along the length of a shaft for any fixed loading condition are plotted to find maximum values to be used to verify or to size the shaft. In general, there are two bending-moments in two different planes. As an example, *Figure 44.2* shows that, although bending takes place in two planes, a vectorial resultant of the two moments (Pythagoras' theorem of *Table IV*) is used in flexure formula since the shaft has a circular cross section. Once the resultant is obtained on the different shaft sections, it is possible to find on which section the bending-moment is maximum. The torque diagram has stepwise trend being absorbed at first by gear C and lastly by second gear B.

The sizing of diameter d of an axle subjected to a bending moment (M_f) is based on determination of the section modulus, as for example $Z_f = \pi d^3/32$ of a full circle (*Table VII*), and the maximum normal stress $\sigma_{max} = M_f/Z_f$ with **20-7**:

$$d_{min} = \left(\frac{32 M_f}{\pi \sigma_{all}}\right)^{1/3} \qquad \textbf{44-1}$$

Table 44.1
Mechanical properties of structural materials

Steels# – The properties of Quenched and Tempered (Q&T) steels are related to a reference section with $d \leq 16$ mm diameter or $t \leq 8$ mm thickness##. The modulus of elasticity of Q&T steels is $E = 200 \div 206$ GPa. With the exception of particular applications, the use of steels marked with asterisks (as a function of stress level) is suggested.

Q&T steel	*Designation*	*Tensile strength* (R_m or σ_u) [MPa]	*Upper yield strength* (R_e or σ_y) [MPa]	*Elongation* A [%]
Carbon	C 25 (*)	550 ÷ 700	370	19
	C 30	600 ÷ 750	400	18
	C 35	630 ÷ 780	430	17
	C 40 (*)	650 ÷ 800	460	15
	C 45	700 ÷ 850	490	13
	C 50 (*)	750 ÷ 900	520	13
	C 55	800 ÷ 950	550	12
	C 60	850 ÷ 1000	580	11
Mn alloy	28 Mn 6	800 ÷ 950	590	13
Cr alloy	41 Cr 4	1000 ÷ 1200	800	11
Cr-V alloy	51 Cr V 4	1100 ÷ 1300	900	9
Cr-Mo alloy	25 Cr Mo 4	900 ÷ 1100	700	12
	34 Cr Mo 4 (**)	1000 ÷ 1200	800	11
	42 Cr Mo 4	1100 ÷ 1300	900	9
Cr-Ni-Mo alloy	36 Cr Ni Mo 4	1100 ÷ 1300	900	10
	34 Cr Ni Mo 6	1200 ÷ 1400	1000	9
	30 Cr Ni Mo 8	1250 ÷ 1450	1050	9
	36 Cr Ni Mo 16 (***)	1250 ÷ 1450	1050	9

#) The mechanical properties of rolled steels are shown in *Table 45.1*.
##) Tensile strength and upper yield strength decrease continuously passing from reference section with $d \leq 16$ mm diameter or $t \leq 8$ mm thickness of this Table to reference sections having $16 < d \leq 40$ mm or $8 < t \leq 20$ mm and then to reference sections having $40 < d \leq 100$ mm or $20 < t \leq 60$ mm.
(*) Steels suggested for current application.
(**) Steel suggested for high level stresses.
(***) Steel suggested for extremely high level stresses.

Table 44.1 (continued)

Gray cast iron – Reference is made to a casting diameter $d = 30$ mm; number in bold type indicates the minimum acceptance tensile strength. The yield strength for the gray cast iron is in general determined as the stress required to produce a permanent strain equal to 0.1% and is variable between 65 and 80% of tensile strength. Elongation is $A = 0.8 \div 0.3\%$.

Designation	*Tensile strength* (R_m or σ_u) [MPa]	*Compressive strength* [MPa]	*Modulus of elasticity E* [GPa]
EN-GJL-150	**150** ÷ 250	665	78 ÷ 103
EN-GJL-200	**200** ÷ 300	790	88 ÷ 113
EN-GJL-250	**250** ÷ 350	910	103 ÷ 118
EN-GJL-300	**300** ÷ 400	990	108 ÷ 137
EN-GJL-350	**350** ÷ 450	1030	123 ÷ 143

Ductile (nodular, spheroidal) cast iron – The compressive strength is considered equal to 2 times of tensile strength.

Designation	*Tensile strength* (R_m or σ_u) [MPa]	*Off-set yield stress* ($R_{p0.2}$) [MPa]	*Minimum elongation A* [%]	*Modulus of elasticity E* [GPa]
EN-GJS-350-22	350	220	22	169
EN-GJS-400-15	400	250	15	169
EN-GJS-500-7	500	320	7	169
EN-GJS-600-3	600	370	3	174
EN-GJS-700-2	700	420	2	176
EN-GJS-800-2	800	480	2	176

Sizing requires that the maximum stress σ_{max} is equal to the allowable stress σ_{all} (**19-13**):

$$\sigma_{max} = \frac{M_f}{(Z_f)_{min}} = \frac{M_f}{\pi d_{min}^3/32} = \frac{32M_f}{\pi d_{min}^3} = \sigma_{all} \Rightarrow d_{min}^3 = \frac{32M_f}{\pi\sigma_{all}} \Rightarrow d_{min} = \left(\frac{32M_f}{\pi\sigma_{all}}\right)^{1/3}$$

In case of shafts acted upon a pure torque (M_t), i.e. when the bending moment is negligible, the torsional sizing of diameter d is based on determination of the polar section modulus, as for example $Z_t = \pi d^3/16$ for a solid round section (**20-11′**), and of the maximum shear stress with the equation **20-11** ($\tau_{max} = M_t/Z_t$):

$$d_{min} = \left(\frac{16M_t}{\pi\tau_{all}}\right)^{1/3} \qquad \textbf{44-2}$$

$$\tau_{max} = \frac{M_t}{(Z_t)_{min}} = \frac{M_t}{\pi d_{min}^3/16} = \frac{16M_t}{\pi d_{min}^3} = \tau_{all} \Rightarrow d_{min}^3 = \frac{16M_t}{\pi\tau_{all}} \Rightarrow d_{min} = \left(\frac{16M_t}{\pi\tau_{all}}\right)^{1/3}$$

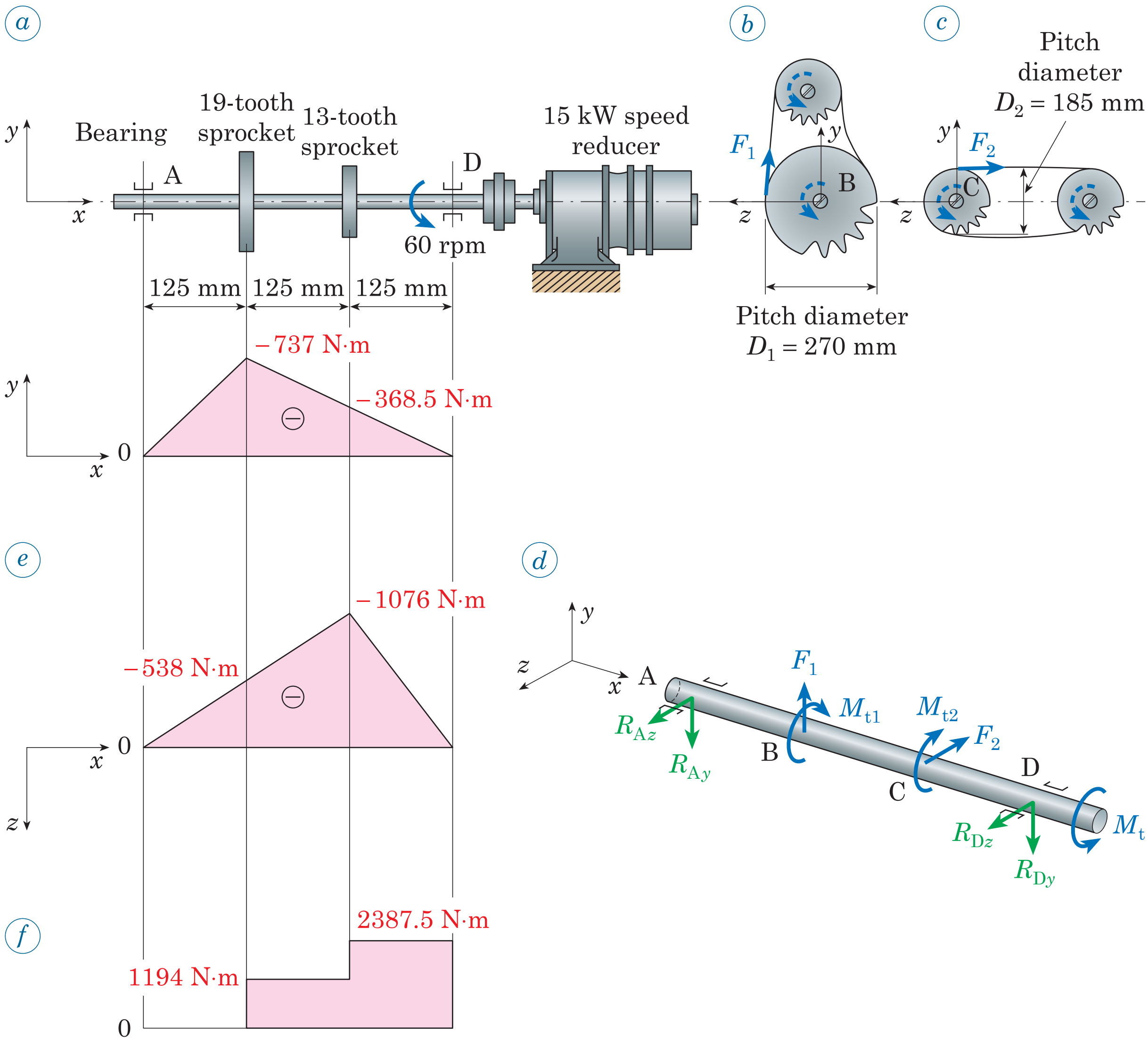


Fig. 44.2 - Bending moments and torque diagrams of a solid steel shaft driving two sprocket wheels with roller chains (from *Figure 21.9*); similar sprockets and roller chains are commonly used on bicycles. Calculations aimed to shaft verification are shown in *Example 21.4*.

When shafts are subjected to combined torsional and bending loads, the combined bending and torsion are taken into account by the equivalent (*ideale*) bending moment $M_{f,id} = \sqrt{M_f^2 + 0.75 M_t^2}$. $M_{f,id}$ substitutes the bending moment M_f in the equation **44-1**, thus obtaining:

$$d_{min} = \left(\frac{32 M_{f,id}}{\pi \sigma_{all}}\right)^{1/3} \qquad \textbf{44-3}$$

If the mechanical element, instead to be designed, has to be verified, the Von Mises [*ideale*] stress σ_{id} (**19-11**) that, in the most general case of bending and torsional loading, takes into account the normal stress given by **20-7** ($\sigma_{max} = M_f/Z_f$) and the shear stress given by **20-11** ($\tau_{max} = M_t/Z_t$), shall be equal or less than the allowable stress σ_{all}:

$$\sigma_{id} = \sqrt{\sigma^2 + 3\tau^2} \leq \sigma_{all} \qquad \textbf{44-4}$$

44.1.3 Rigidity

The preceding paragraph dealt with the problem of designing a shaft diameter d able to meet a given stress. But a shaft so designed may still be unsatisfactory because it lacks rigidity (or stiffness). Insufficient rigidity (*Card 44.1*) can result in poor performance of various shaft-mounted elements such as gears, clutches, bearings, etc. and can determine a much greater diameter than that calculated with the previous equations. As an example, in a transmission, the gears have to be supported by a rigid shaft; if the shaft bends too much, the teeth will not mesh properly, and the result will be excessive impact, noise, wear, and early failure (*Figure 44.3*).

Therefore limits must be introduced on the slope and the maximum linear deflection of a shaft (see y_{max} in *Appendix B.2*). If the modulus of elasticity E of the shaft material and its area moment of inertia I are known, the absolute values of the slope θ and the maximum linear deflection y_{max} versus load (F) occurring in the middle ($l/2$) are given by (*Paragraph 23.1*):

$$\theta_A = \frac{Fl^2}{16EI} \quad \textbf{(23-3)} \qquad y_{max} = \frac{Fl^3}{48EI} \quad \textbf{(23-4)} \qquad \textbf{44-5}$$

while, when the load is applied in whatever point B of the beam, the absolute value of linear deflection $y_{max} = y_B$ is:

$$y_{max} = \frac{Fa^2b^2}{3EIl} \qquad \textbf{44-6}$$

From *Appendix B.2-6*:

$$y_{AB} = \frac{Fbx}{6EIl}\left(x^2 + b^2 - l^2\right) \Rightarrow y_{max} = y_{AB}\Big|_{x=a} = \frac{Fba}{6EIl}\left(a^2 + b^2 - l^2\right) = \frac{Fba}{6EIl}\left(a^2 + b^2 - a^2 - b^2 - 2ab\right) =$$

$$= -\frac{Fba}{6EIl}\left(2ab\right) = -\frac{Fa^2b^2}{3EIl}$$

Shafting must be proportioned not only to provide the strength required to transmit a given torque, but also to prevent torsional deflection (twisting) through a greater angle than that found satisfactory for a given type of service. By considering the angle of twist θ of two shaft cross-sections at distance l, the angular deflection is related to the ratio θ/l. The limit of this ratio, usually called torsional rigidity because it is related to the product of shear modulus of elasticity (G) and the polar moment of inertia (J) of the shaft, is $\theta/l = M_t/(GJ) \leq 0.25°/\text{m}$ (**20-10**).

Card 44.1

Limits to absolute or relative deformations of rotating shafts

Limit of linear deflection – Maximum allowed deflection y_{max} versus maximum distance l [m] between bearings:

Application	coarse	common	speed reducer	machine tool
Deflection y_{max}	$l/1000$	$l/2000$	$l/(3000 \div 4000)$	$l/(5000 \div 6000)$

Slope: $\theta \leq 0.001$ mrad $= 0.05°^{(*)}$, where θ is the slope due to deflection.

Gear rotation: $\phi_{max} \leq 0.5$ mrad $= 0.03°^{(*)}$, where ϕ_{max} is the maximum rotation due to deflection at the point where gear is located.

Torsional deflection (twisting): $\theta/l \leq 4.4$ mrad/m $= 0.25°/\text{m}^{(*)}$, where θ is the angular deflection of a uniform round bar per meter of length subjected to a torsional moment M_t.

Allowable rotations in rolling bearings – Maximum allowable rotation versus bearing type:

Bearings	ball	straight roller	tapered roller	self-aligning ball
Limit of rotation(*)	3 mrad = 0.17°	0.9 mrad = 0.05°	0.6 mrad = 0.033°	26 ÷ 52 mrad = 1.5 ÷ 3°

(*) in milliradiant [mrad] and degree [°] (*Table I*)

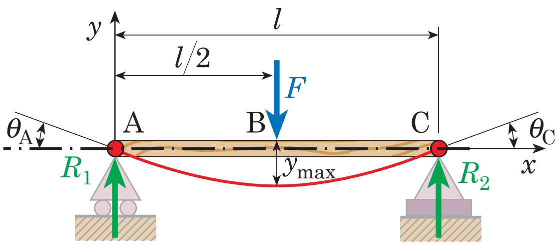


Fig. 44.3-a - Linear deflection y_{max} and slope θ of a deflected shaft.

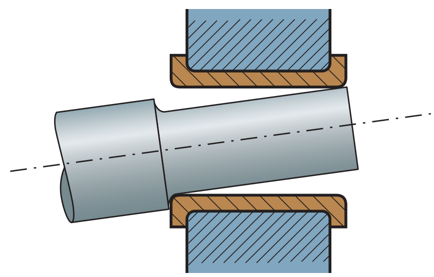


Fig. 44.3-b - Too large deflection of a journal in respect of the bearing.

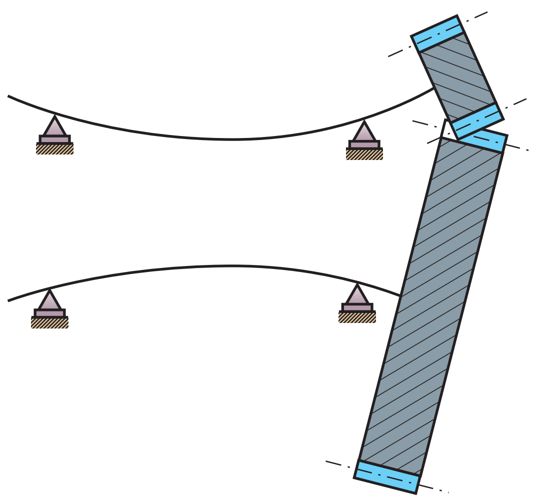


Fig. 44.3-c - If the linear deflection of geared shafts is too large, the life of the gears will be shortened because of additional impact forces during meshing and greater wear of tooth surfaces. As a general rule for lengthy shafts, it is strongly suggested to avoid cantilevered gears requiring a tight meshing.

44.1.4 Shaft keyways and preferred numbers

A *key* is a demountable machinery component that, when assembled into keyseats, provides a positive means for transmitting torque between the shaft and hub. It is used on shafts to secure rotating elements such as gears and pulleys. *Paragraph 45.6* will deal with the sizing of keys, while here (*Table 44.2*) their unified dimensions, as a function of shaft diameter d, are shown to complete the shaft sizing. If t_1 is the keyseat depth on the shaft (and t_2 is that on the hub), the real diameter of the shaft is: $d \geq d_{min} + t_1$ (*Figure 44.4*).

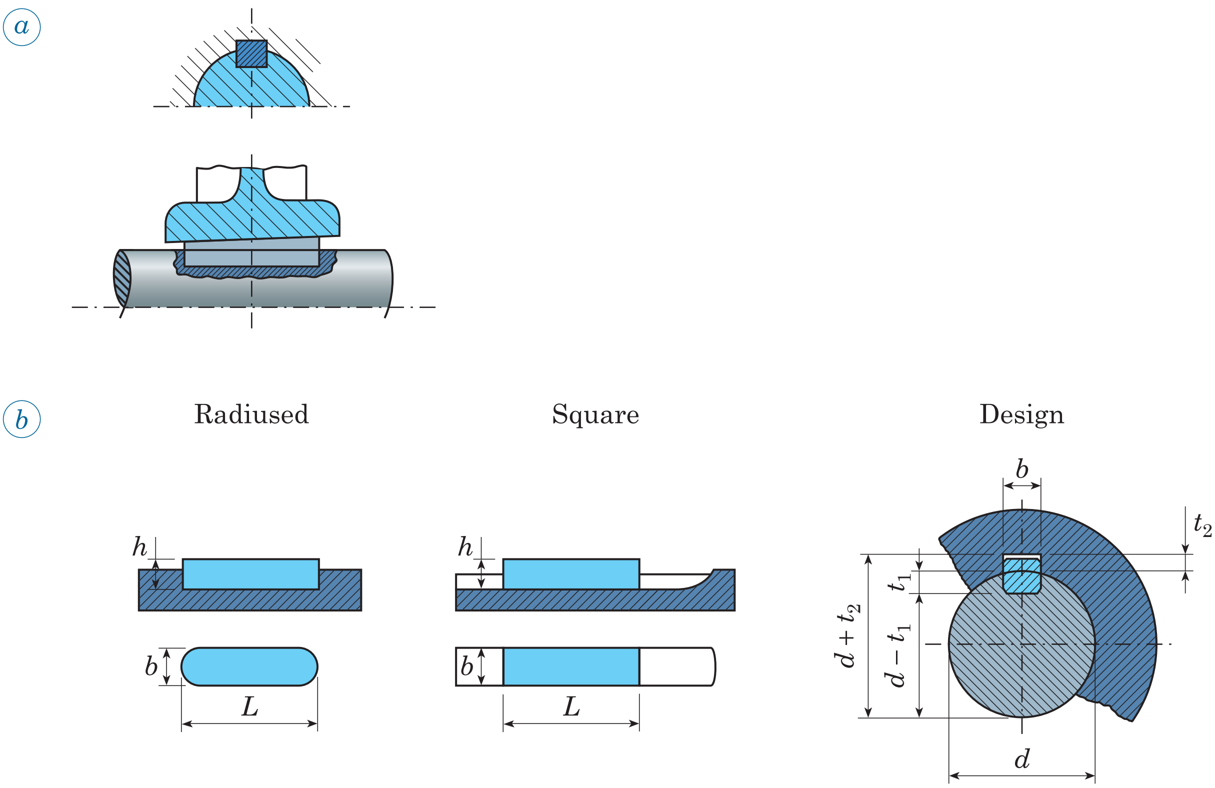


Fig. 44.4 - a) The tapered key (1:100) when assembled into keyseat prevents any relative motion between shaft and hub.

b) The parallel key is a prismatic shaped solid that prevents any rotation between shaft and hub but it allows the axial motion of the hub in respect of the shaft.

The value of the shaft diameter would be finally corrected according to preferred numbers (*Appendix C.1* – UNI 2017, series Ra5, Ra10, Ra20). *Preferred numbers* (also called *preferred values*) are series of numbers selected to be used for standardization purposes in preference to any other number. Their use will lead to simplified practice and they should be employed whenever possible for individual standards sizes and ratings. They serve two purposes:

– to increase the probability of compatibility between objects designed at different times by different people;

Table 44.2
Characteristics sizes of tapered and parallel keys
Width b, heigth h and keyseat depth t_1 on the shaft and t_2 on the hub [mm] versus shaft diameter d.

Shaft diameter d	*Tapered key*		*Keyseat depth*	
	$b \times h$	L	*Shaft* t_1	*Hub* t_2
$6 \div 8$	2×2	$6 \div 20$	1,2	0,5
$> 8 \div 10$	3×3	$6 \div 36$	1,8	0,9
$> 10 \div 12$	4×4	$8 \div 45$	2,5	1,2
$> 12 \div 17$	5×5	$10 \div 56$	3	1,7
$> 17 \div 22$	6×6	$14 \div 70$	3,5	2,2
$> 22 \div 30$	8×7	$18 \div 90$	4	2,4
$> 30 \div 38$	10×8	$22 \div 110$	4,5	2,4
$> 38 \div 44$	12×8	$28 \div 140$	5	2,4
$> 44 \div 50$	14×9	$36 \div 160$	5,5	2,9
$> 50 \div 58$	16×10	$45 \div 180$	6	3,4
$> 58 \div 65$	18×11	$50 \div 200$	7	3,4
$> 65 \div 75$	20×12	$56 \div 220$	7,5	3,9
$> 75 \div 85$	22×14	$63 \div 250$	9	4,4
$> 85 \div 95$	25×14	$70 \div 280$	9	4,4
$> 95 \div 110$	28×16	$80 \div 320$	10	5,4
$> 110 \div 130$	32×18	$90 \div 360$	11	6,4
$> 130 \div 150$	36×20	$100 \div 400$	12	7,1
$> 150 \div 170$	40×22	$110 \div 400$	13	8,1
$> 170 \div 200$	45×25	$125 \div 400$	15	9,1
Shaft diameter d	*Parallel key*		*Keyseat depth*	
	$b \times h$	L	*Shaft* t_1	*Hub* t_2
$6 \div 8$	2×2	$6 \div 20$	1,2	1
$> 8 \div 10$	3×3	$6 \div 36$	1,8	1,4
$> 10 \div 12$	4×4	$8 \div 45$	2,5	1,8
$> 12 \div 17$	5×5	$10 \div 56$	3	2,3
$> 17 \div 22$	6×6	$14 \div 70$	3,5	2,8
$> 22 \div 30$	8×7	$18 \div 90$	4	3,3
$> 30 \div 38$	10×8	$22 \div 110$	4,5	3,3
$> 38 \div 44$	12×8	$28 \div 140$	5	3,3
$> 44 \div 50$	14×9	$36 \div 160$	5,5	3,8
$> 50 \div 58$	16×10	$45 \div 180$	6	4,3
$> 58 \div 65$	18×11	$50 \div 200$	7	4,4
$> 65 \div 75$	20×12	$56 \div 220$	7,5	4,9
$> 75 \div 85$	22×14	$63 \div 250$	9	5,4
$> 85 \div 95$	25×14	$70 \div 280$	9	5,4
$> 95 \div 110$	28×16	$80 \div 320$	10	6,4
$> 110 \div 130$	32×18	$90 \div 360$	11	7,4
$> 130 \div 150$	36×20	$100 \div 400$	12	8,4
$> 150 \div 170$	40×22	$110 \div 400$	13	9,4
$> 170 \div 200$	45×25	$125 \div 400$	15	10,4

Standard length L: 6; 8; 10; 12; 14; 18; 20; 22; 25; 28; 32; 36; 40; 45; 50; 56; 63; 70; 80; 90; 100; 125; 140; 160; 180; 200; 220; 250; 280; 320; 360; 400 mm.

– the preferred numbers are chosen such that when a product is manufactured in many different sizes, these will end up roughly equally spaced on a logarithmic scale. They therefore help to minimize the number of different sizes that need to be manufactured or kept in stock.

Examples of applications of preferred numbers are as follows:

– characteristic dimensions, such as diameters and lengths, areas, volumes, weights, capacities;

– ratings of machinery and apparatus in power, voltage, current, speed, pressure, heat, etc.;

– characteristic ratios of figures for all kinds of units.

Example 44.1 Torsion of a shaft

Verify the limit of the torsional rigidity θ/l of a steel shaft (modulus of elasticity $E = 206$ GPa and Poisson ratio $\nu = 0.3$) having a solid circular section with diameter $d = 40$ mm and transmitting the power $P = 5.0$ kW at the rotational speed $n = 600$ rev/min (rpm).

SOLUTION

The limit of torsional rigidity θ/l is (*Card 44.1*) $\theta/l \leq 4.4$ mrad/m = 0.25°/m, where mrad means milliradiant. The rotation per meter θ/l due to the torsion is given by (**20-10**) $\theta = (M_t l)/(GJ) \;\Rightarrow\; \theta/l = M_t/(GJ)$, where:

– the torque M_t [N·m] is a function (**20-12**) of P and n;

– the shear modulus of elasticity G is obtained from the elastic constants relationship (**19-7**) $E = 2G(1+\nu)$;

– the polar moment of inertia $J = \pi d^4/32$ (*Table VII*).

$$M_t \approx 9.55\frac{P}{n} = 9.55 \times \frac{5000\ \text{W}}{600\ \text{rpm}} = 79.6\ \text{N·m}$$

$$E = 2G(1+\nu) \;\Rightarrow\; G = \frac{E}{2(1+\nu)} = \frac{206\ \text{GPa}}{2 \times (1+0.3)} = 79.2\ \text{GPa}$$

$$J = \frac{\pi d^4}{32} = \frac{\pi \times (0.04\ \text{m})^4}{32} = 2.51 \times 10^{-7}\ \text{m}^4$$

$$\frac{\theta}{l} = \frac{M_t}{GJ} = \frac{79.6\ \text{N·m}}{79.2 \times 10^9\ \text{Pa} \times 2.51 \times 10^{-7}\ \text{m}^4} = 4.0 \times 10^{-3}\ \text{rad/m} = 4.0\ \text{mrad/m} = 0.23°/\text{m} < 0.25°/\text{m}$$ ◀

44.2 Bearings

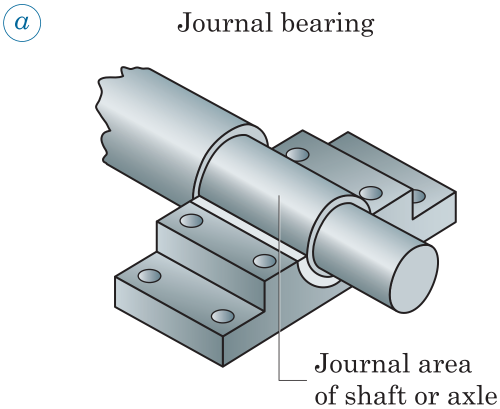
44.2.1 Definitions

The shaft needs a good support to ensure stability and frictionless rotation. In a sleeve bearing, a part of the shaft (*journal*) rotates or oscillates within a sleeve (*bearing*) and the relative motion is sliding (*Figure 44.5*). In an *antifriction bearing*, the main relative motion is rolling. For example, gear teeth mate with each other by combination of rolling and sliding. Journals are classified according to location as *end-wise* (i.e. near the ends of the shaft) and *intermediate journals*. Bearings, that provide sliding contact between mating surfaces, fall into three general classes (*Figure 44.6*): *radial bearings* that support rotating shafts or journals, *thrust bearings* that support axial load on rotating members, and *guide* or *slipper bearings* that guide moving parts according to a straight line. The most usual type of radial bearings are the plain full journal bearing with 360° contact with its mating journal and the partial journal bearing, which has less than 180° contact. This latter type is used when the load direction is constant bringing to sensible advantages of simplicity, ease of lubrication, and reduced frictional loss.

44.2.2 Journal bearings

The relative motion between the parts of plain bearings may take place:

– as pure sliding without the benefit of a liquid or gaseous lubricating medium between the moving surfaces such as in dry-film conditions when using nylon or Teflon;
– under hydrodynamic conditions with a thick-film build-up of lubricating medium, with either whole or partial separation of the bearing surfaces;
– with hydrostatic lubrication where the lubricant is introduced under pressure between the mating surfaces causing a force opposite to the applied load and the lifting or separation of these surfaces;
– with mixed-film lubrication, when the bearing operates partly under hydrodynamic conditions and partly under dry- or thin-film conditions.



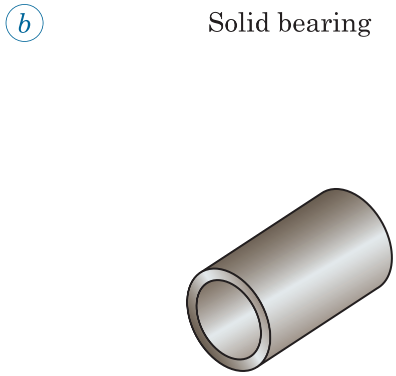


Fig. 44.5 - Journal bearing (a) and solid bearing (b).

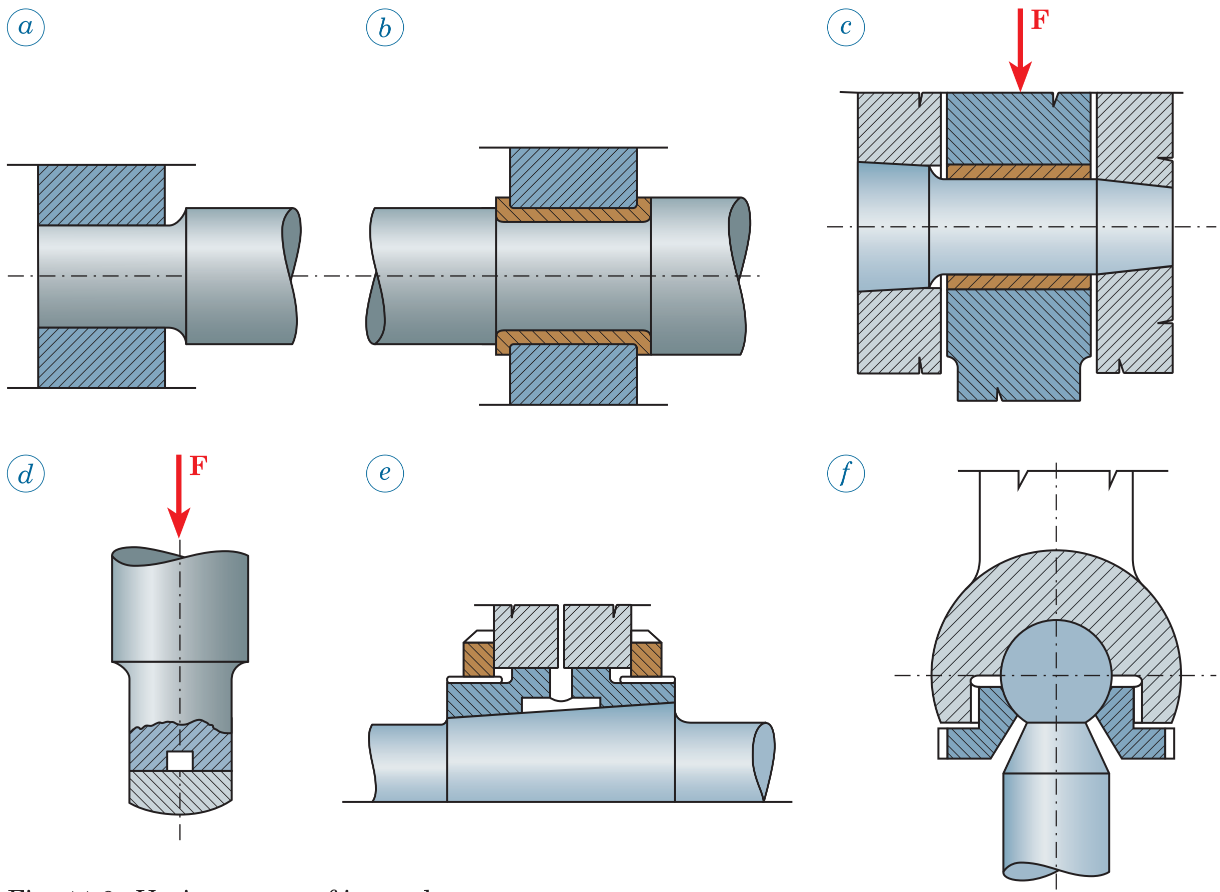


Fig. 44.6 - Various types of journals.
a) End-wise journal.
b) Intermediate journal.
c) Radial sliding bearing, more commonly called sleeve bearing.
d) Thrust bearing.
e) Tapered journal.
f) Ball-and-socket joint.

Thus the lubrication is able to reduce friction, wear, and heating of machine parts which move relative to each other.

Figure 44.7 shows the operation scheme of *journal bearings* and the characteristic dimensions l and d of an end-wise journal. Materials used in journal bearings are characterized by low friction coefficients (sintered iron, bronze, antifriction alloys based on tin and lead); if not sufficiently able to bear loads, they are stratified on a more resistant material. The friction coefficient of the material can be improved with a heat treatment (carbonitriding, sulfinization, etc.). The most sensible improvement of the friction coefficient is due to lubrication with oil that, inter alia, gives a substantial contribution to reduce wear and heating. In order to control the temperature increase of the film of lubricant between the bearing moving surfaces, it goes often to the forced oil feeding with a pump or with a design favoring the oil flow rate, as for example the splash lubrication.

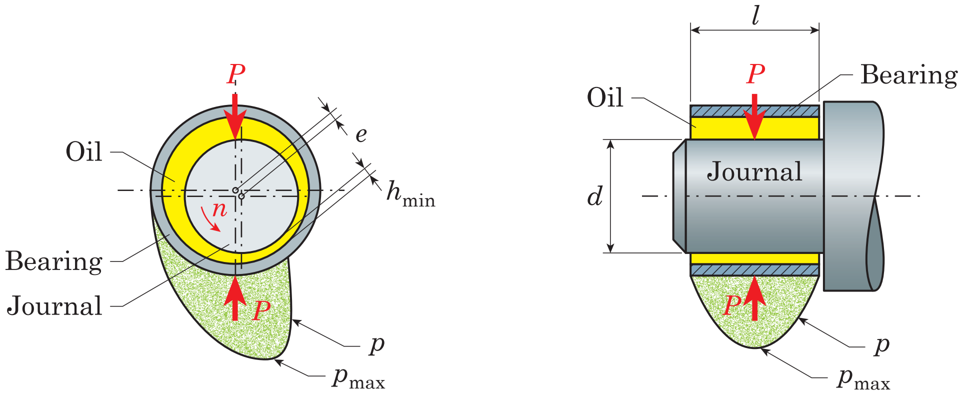


Fig. 44.7-a - Thick-film lubrication (hydrodynamic conditions) pressure distribution and film thickness. The journal bearing can be considered as a rotoidal pair (see *Paragraph 24.1.1*) represented by the journal rotating in the bearing. The pair can work if there is a radial clearance (difference in the radii of the bearing and journal) between journal and seat. At high rotational speed n, the rotating journal pumps the lubricant around the bearing into a wedge-shaped space and the hydrodynamic lubrication takes place with a pressurized thick-film separating the two surfaces of the journal and the bearing. The pressure distribution p is used to equilibrate the load P. The figure shows the journal diameter d, the pressure distribution with its maximum p_{max}, the minimum oil film thickness h_{min} and the eccentricity e (distance between the center of the bearing and the center of the shaft).

Fig. 44.7-b - Under hydrodynamic conditions, the loaded surfaces are separated by a lubricant thick film able to avoid the direct metal-to-metal contact. Passing from the nonrotating journal (on the left) with the load P' acting upon its center to the rotating journal (on the right), the applied load P can be supported via hydrodynamic lubrication.

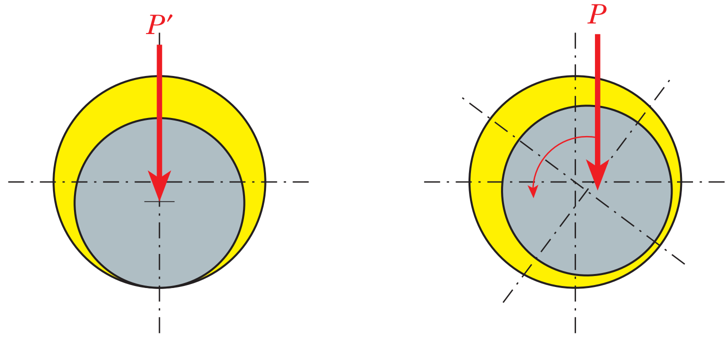
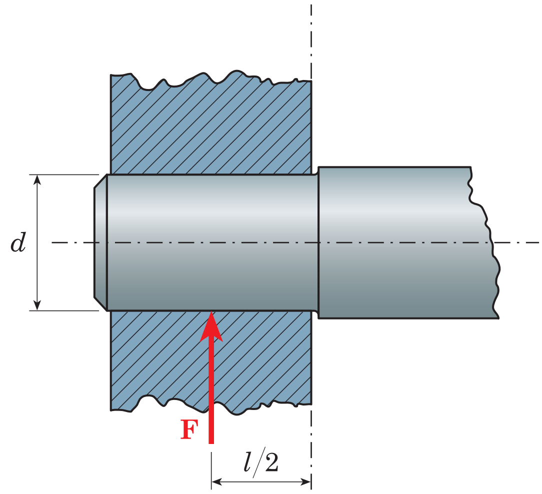


Fig. 44.7-c - End-wise journal; d is the minimum diameter and l is the length.



The journal bearings can be divided in:

– *bushes* (*Figure 44.8*) of soft material, like brass or gun metal, are provided and the body or main block is made of cast iron. Bush is a hollow cylindrical piece which is fitted in a housing to accommodate the mating part. When the bush gets worn out, it can be easily replaced. The outside of the bush is a driving fit (interference fit) in the hole of the casting whereas the inside is a running fit for the shaft. The bearing material used may be white metal (Babbit – tin/Cu/lead/antimony), copper alloy (brass, gunmetal) or aluminum alloy. In case of dry-film conditions, the hollow cylinder is made up of a non-metallic material;
– self-lubricating *sintered bushes* with lubricant, within the porous surface pits, and a maximum peripheral speed $v = 5$ m/s;
– *main crank shaft and conrod bearings* of internal combustion engines made up of semi-cylindrical steel shells covered by antifriction material and able to work with fluid lubricant till to $v = 40$ m/s.

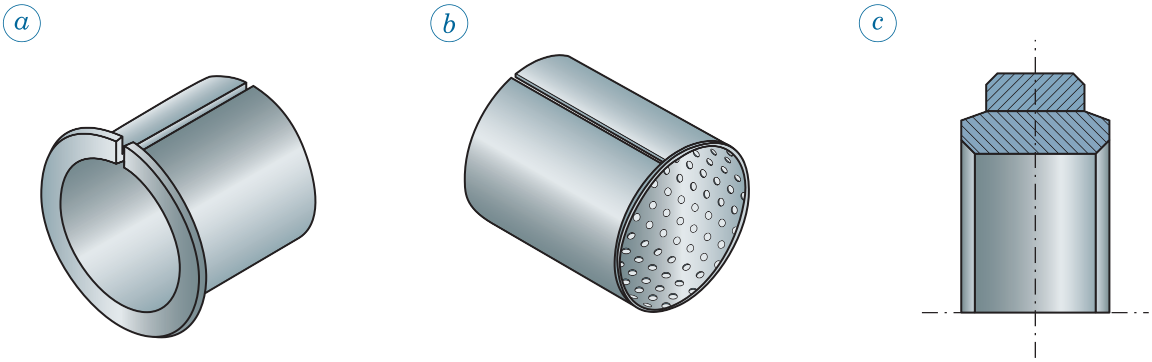


Fig. 44.8 - Various types of bushes (usually as one piece, but in two halves to make their mounting possible):
a) with flange;
b) cylindrical with synthetic material;
c) ball joint.

The bush life is determined by reading on manufacturer's catalogue the basic load ratings (dynamic C and static C_0) and other parameters as a function of principal dimensions as bearing bore and outside diameter, and length. This approach will be examined in details when speaking of rolling bearings.

The diameter d of the *end-wise journal* is (*Figure 44.7-c*) a function of the radial force F, as a sum of forces acting on more planes, and the length to diameter ratio l/d of the journal. Knowing F and the allowable stress of the material σ_{all}, l/d is read on *Table 44.3* as a function of the type of application; then the diameter d and afterwards the journal length $l = d \cdot (l/d)$ are determined.

$$d = \sqrt{\frac{5F}{\sigma_{all}}\left(\frac{l}{d}\right)} \tag{44-7}$$

The journal is considered as a uniform loaded cantilever of length l; the uniform load is substituted by a center load F applied at $l/2$. By neglecting the shear stress, the maximum normal stress due to bending is (**20-7**) $\sigma_{max} = M_f/Z_f$, where the section modulus of a full circle (*Table VII*) is $Z_f = \pi d^3/32$. By equalizing the maximum normal stress to the allowable stress (**19-13**), the diameter d is obtained:

$$\sigma_{max} = \frac{M_f}{Z_f} = \frac{F\,l/2}{\pi d^3/32} = \frac{16}{\pi}\frac{Fl}{d^3} \approx 5\frac{F}{d^2}\frac{l}{d} = \sigma_{all} \quad \Rightarrow \quad d^2 = \frac{5F}{\sigma_{all}}\frac{l}{d} \quad \Rightarrow \quad d = \sqrt{\frac{5F}{\sigma_{all}}\frac{l}{d}}$$

The diameter d of the *intermediate journal* (*Figure 44.6-b*), subjected to combined bending and torsional loading, is calculated by applying the same procedure as that of the shafts: the equivalent bending moment $M_{f,id}$ and then the diameter are determined with **44-3**. Once known d, the journal length $l = d \cdot (l/d)$ is obtained by reading on *Table 44.3* the l/d ratio as a function of the type of application.

Table 44.3
Journal bearing
Most frequent average values of l/d ratio of a journal-bearing pair. Lower l/d values accompany greater diameters because in such a case the journal is more rigid.

Application	l/d
Automotive	0.5 ÷ 1.0
Electric machines	0.8 ÷ 1.5
Steam turbines	1.0 ÷ 1.2
Transmissions	1.2 ÷ 1.5
Lifting apparatus	1.2 ÷ 1.5
Machine tools	1.2 ÷ 2.0

Once known the journal diameter, its bearing capacity has to be verified by specifying that (*Figure 44.9*) the *average contact pressure* $p = F/(dl)$ is less than or equal to the allowable pressure p_{all} of *Table 44.4*. Instead of p_{all}, the maximum surface velocity of journal relative to bearing surface v_{max} [m/s] can be considered. The Table shows also the factor $(pv)_{limit}$ (average pressure p [N/mm^2] multiplied by the peripheral shaft velocity v [m/s]) that is usually related to two important limiting criteria for acceptable bearing performance: operating temperature and wear rate. The $(pv)_{limit}$ factor can be used as an alternative or as an addition to the heating factor W that will be introduced by the equation **44-9**.

$$p = \frac{F}{dl} \le p_{all} \qquad\qquad pv \le (pv)_{limit} \qquad\qquad \textbf{44-8}$$

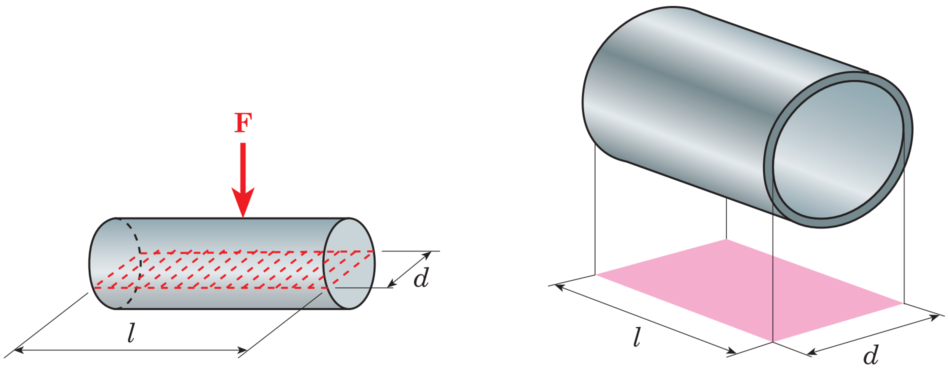


Fig. 44.9 - The projected area, interested by the average contact pressure p relative to the load F, is given by the product of journal diameter d for its length l.

Table 44.4
Parameters of journal bearings
Allowable average pressure p_{all} [N/mm²] and factor $(pv)_{limit}$ $\left[\left(N/mm^2\right) \cdot (m/s)\right]$ of the journal-bearing pair. Being (*Paragraphs 1.13* and *1.14*) the work w measured as joule J [J = Nm] and the power P measured as watt W $\left[W = (Nm)/s\right]$, pv can be represented by W/mm².

Application and peripheral velocity v [m/s]	*Bearing material*	p_{all} [N/mm²]	$(pv)_{limit}$ $\left[(Nm)/(mm^2 s)\right]$
Load bearing journals, $v < 1$	Composite	< 5.0	0.1 ÷ 1.7
Transmission, rotary pump, $v < 6$	Antifriction	1.5 ÷ 2.0	2.0 ÷ 3.5
Turbomachines, $v < 60$	Antifriction	0.5 ÷ 2.0	20 ÷ 25
Machine tools	Antifriction or bronze	3.0 ÷ 9.0	4.0 ÷ 8.0
Lifting apparatuses	Bronze	7.0 ÷ 13.0	12 ÷ 20
Electric machines, household appliances	Antifriction	0.5 ÷ 1.2	5 ÷ 12
Otto-cycle engines – spindle – conrod small end – conrod big end – crank – main bearings	 Bronze Antifriction or bronze Antifriction or bronze	 25 ÷ 30* 4.0 ÷ 10* 9.0 ÷ 15*	 90 ÷ 150* 90 ÷ 150*
Diesel-cycle engines – spindle – conrod small end – conrod big end – crank – main bearings	 Bronze Antifriction or bronze Antifriction or bronze	 25 ÷ 60* 4.0 ÷ 6.0* 4.0 ÷ 15*	 130 ÷ 150* 40 ÷ 140*

(*) Values are calculated at maximum pressure under steady-state operations.

Journals should be also verified, particularly in case of end-wise journals rotating at high speed ($n > 15$ rps = 900 rpm), from the point of view of the heat dissipation. In fact, by conducting away heat, the oil flow provides an important cooling function to prevent overheating of the bearing. Then the temperature increase because of heating has not to be so high to prejudice the lubricant capacity to reduce in a significant way the friction coefficient. The heating verification is conducted with the W factor, that is an indicator of the bearing temperature rise relative to the ambient temperature. This factor should be measured as W/mm^2, but it is usually expressed by the units of *Table 44.5*.

$$W = \frac{Fn}{l} \leq W_{all} \qquad \textbf{44-9}$$

where:

W = heating factor [(N/mm)·rpm];

W_{all} = allowable heating factor [(N/mm)·rpm];

F = total force on journal bearing [N];

n = rotational speed [rpm];

l = journal length [mm].

Table 44.5
Heating factor
Allowable heating factor W_{all} of the journal-bearing pair.

Operations	W_{all} $\left[(\text{N/mm})\cdot\text{rpm}\right]$
Accurate tooling, poor lubrication, no air movement	15,000 ÷ 20,000
Accurate tooling, plentiful lubrication, no air movement	30,000 ÷ 40,000
Accurate tooling, plentiful lubrication, air movement	60,000 ÷ 70,000
Accurate tooling, plentiful lubrication, high air movement	100,000 ÷ 200,000
Accurate tooling, force-feed lubrication with artificial cooling	150,000 ÷ 300,000

Both wear and heating factor depend on pv factor (the pressure p multiplied by the peripheral shaft velocity v) as follows. The sliding block of *Figure 44.10*, subjected to the force F, moves along a plate with contact pressure p [N/mm^2] acting over area A [mm^2], in presence of the coefficient of sliding friction μ [-]. The work [J = Nmm] done by the force μpA [N] during displacement $s = vt$ [mm], where v [mm/s] is the sliding velocity and t [s] is time, is $(\mu pA)(vt)$. The material volume [mm^3] removed due to *linear sliding wear* w [mm] is wA and is proportional (~) to the work done: $wA \sim \mu pAvt$. By introducing the proportionality factor K, which includes μ and is determined from laboratory testing, and two other factors accounting for departures from the laboratory conditions under which K was measured: f_1 depending on motion type, load, and speed, and f_2 to account for temperature and cleanliness conditions, the linear wear rate w/t or the linear wear w can be calculated. It results that the wear w is a function of pv; this product is measured as W/mm^2, being a power [N·mm/s] divided by an area [mm^2].

$$\frac{w}{t} = f_1 f_2 Kpv \qquad \text{or} \qquad w = f_1 f_2 Kpvt \qquad \textbf{44-10}$$

The W factor formula **44-9** can be obtained as a power balance. The power P [W], as a function (**6-13**) of journal rotational speed n [rev/min] and its diameter d [m], is generated (**7-10**) by the friction force μF [N] multiplied by the peripheral shaft velocity v [m/s]:

$$P = (\mu F)v = (\mu F)\omega r = (\mu F)\frac{2\pi n}{60}\frac{d}{2} = \mu F\frac{\pi n d}{60} \qquad \textbf{44-11}$$

This dissipated power produces a heat rate $\dot{Q}$ [W] depending on (**27-5**):

- convection heat transfer coefficient h [W/(m^2·K)];
- journal lateral area $A = \pi dl$ exchanging heat with surroundings;
- lubricant temperature increase ΔT [K] with respect to the ambient temperature.

By collecting the term $(Fn)/l$, the heating factor W is obtained:

$$P = \dot{Q} \quad \Rightarrow \quad \mu F\frac{\pi n d}{60} = hA\Delta T = h\pi dl\Delta T \quad \Rightarrow \quad \frac{\mu F n}{60} = hl\Delta T \quad \Rightarrow \quad \frac{Fn}{l} = \frac{60h\Delta T}{\mu} = W \qquad \textbf{(44-9)}$$

If the force F and the rotational speed n are expressed (**44-8**) as a function of the pressure p (**44-8**) and the peripheral velocity v (**6-13**) respectively, the heating factor $W = (Fn)/l$, when using coherent units and unless π, equals the product pv:

$$W = \frac{Fn}{l} = \frac{(pdl)\dfrac{v}{\pi d}}{l} = \frac{1}{\pi}pv \qquad \textbf{44-12}$$

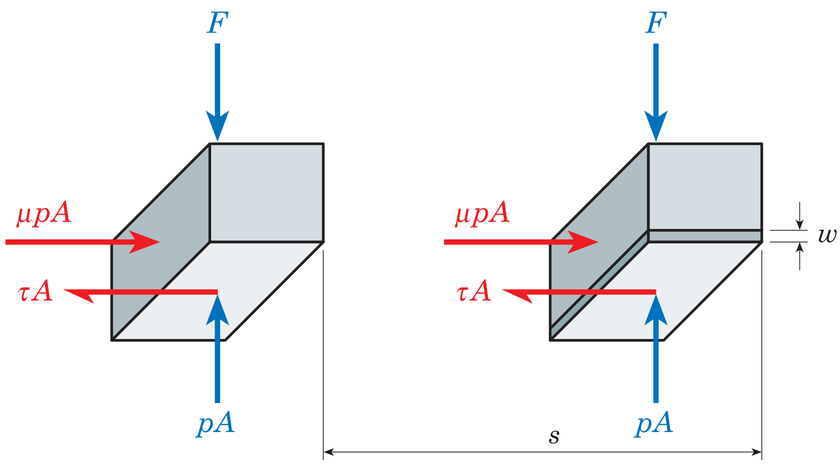


Fig. 44.10 - Sliding block moving along a plate with μ as a friction coefficient and subjected to linear wear w during the displacement s.

In case of thrust bearings, end-wise (*Figure 44.11-a*) or intermediate configuration (*Figure 44.11-b*), the annular surface contact has area $A = \left[\pi\left(D^2 - d^2\right)\right]/4$ (end-wise) and $A = z\left[\pi\left(D^2 - d^2\right)\right]/4$ (intermediate with z runners). The procedure to verify the allowable contact pressure p_{all} (*Table 44.6*) or the heating is the same as that of load bearing journals, but the mean diameter $d_m = (D + d)/2$ instead of journal diameter d is considered; then the peripheral velocity $v = \omega(d_m/2) = \omega(D + d)/4$ is referred to d_m.

$$p = \frac{F}{A} = \frac{4F}{\pi(D^2 - d^2)} \le p_{\text{all}} \qquad W = \frac{Fn}{\frac{D - d}{2}} \le W_{\text{all}} \tag{44-13}$$

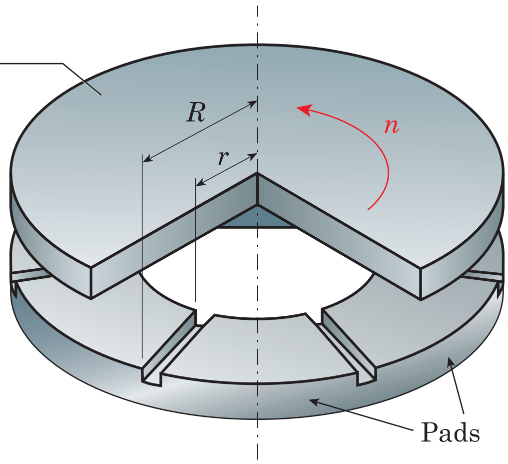


Fig. 44.11-a - Fixed-pad thrust bearing.

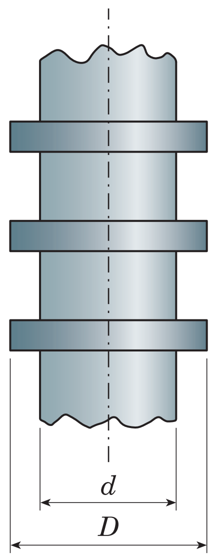


Fig. 44.11-b - Intermediate thrust bearing with more runners (annular collars with diameter D) on a shaft diameter d: $D = (1.5 \div 2.0)d$.

Once fixed the relationship between the inside diameter d and the outside diameter D, usually $d = (0.2 \div 0.4) \cdot D$, for thrust bearings, end-wise (in the case of intermediate bearings see *Figure 44.11-b*) the diameters are calculated as a function of a given value of the allowable pressure p_{all} of *Table 44.6* for the given type of journal and for the given value of peripheral velocity v. Afterwards, the heating is verified by taking $W_{\text{all}} = 30{,}000 \div 70{,}000\,(\text{N/mm}) \cdot \text{rpm}$, where the highest values are referred to force-feed lubrication.

The W factor calculation is analogous to that of load bearing journals (**44-9**) with the only difference that the heat transfer area $A = \left[\pi\left(D^2 - d^2\right)\right]/4$ and the mean diameter $d_m = \left(D + d\right)/2$ are considered. By setting the friction dissipated power equal to the heat flow rate, W is obtained.

$$P = \left(\mu F\right)\frac{2\pi n}{60}\frac{d_m}{2} = \mu F\frac{\pi d_m n}{60}$$

$$\dot{Q} = hA\Delta T = h\pi\frac{\left(D^2 - d^2\right)}{4}\Delta T = h\pi\frac{\left(D + d\right)}{2}\frac{\left(D - d\right)}{2}\Delta T = h\pi d_m\frac{\left(D - d\right)}{2}\Delta T$$

$$P = \dot{Q} \quad\Rightarrow\quad \mu F\frac{\pi d_m n}{60} = h\pi d_m\frac{\left(D - d\right)}{2}\Delta T \quad\Rightarrow\quad \frac{Fn}{\frac{D - d}{2}} = \frac{60h\Delta T}{\mu} = W$$

Table 44.6
Thrust bearings
Allowable average pressure of thrust bearings.

Type and peripheral velocity v [m/s]	*Bearing material*	p_{all} [N/mm^2]
Annular, $v < 10$	Antifriction or bronze	$0.4 \div 0.8$
	Antifriction/bronze (boundary lubrication)	< 7.0
Series of collars	Bronze	$0.2 \div 0.9$
Precision machined inclined segments	Antifriction	$1.0 \div 3.0$
Swingable segments, $v < 60$	Antifriction	$2.0 \div 5.0$

Example 44.2 End-wise journal

The end-wise journal of *Figure 44.12* has the rotational speed $n = 130$ rpm and is subject to the force $F = 3.0$ kN. The bush is made of composite material having the maximum peripheral velocity $v_{max} = 0.5$ m/s and the product $(pv)_{limit} = 1.6$ W/mm^2. Size the journal, by using a low value of allowable fatigue stress $\sigma_{all,f} = 24$ N/mm^2, and verify $v_{max} = 0.5$ m/s and $(pv)_{limit}$.

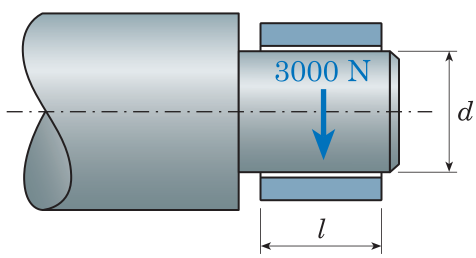


Fig. 44.12 - End-wise journal of *Example 44.2*.

SOLUTION

At first the journal diameter d is calculated with equation **44-7**, by imposing a quite general ratio $l/d = 1$ (*Table 44.3*) because the application is not specified. After calculating the average contact pressure p and the peripheral velocity v, the maximum velocity v_{max} (instead of the allowable pressure p_{all}) and $(pv)_{limit}$ are verified.

$$\textbf{44-7}:\ d = \sqrt{\frac{5F}{\sigma_{all}}\frac{l}{d}} = \sqrt{\frac{5 \times 3000\ \mathrm{N}}{24\ \mathrm{N/mm^2}} \times 1} = 25\ \mathrm{mm} \quad \Rightarrow \quad \frac{l}{d} = 1 \quad \Rightarrow \quad l = d = 25\ \mathrm{mm}$$

$$p = \frac{F}{dl} = \frac{3000\ \mathrm{N}}{25\ \mathrm{mm} \times 25\ \mathrm{mm}} = 4.8\frac{\mathrm{N}}{\mathrm{mm^2}}$$

$$\textbf{6-13}:\ v = \frac{\pi d n}{60} = \frac{\pi \times 0.025\ \mathrm{m} \times 130\ \mathrm{rpm}}{60\ \mathrm{s/min}} = 0.17\ \mathrm{m/s} < v_{max} = 0.5\ \mathrm{m/s}$$

$$\textbf{44-8}:\ pv = 4.8\frac{\mathrm{N}}{\mathrm{mm^2}} \times 0.17\frac{\mathrm{m}}{\mathrm{s}} = 0.816\frac{\mathrm{N}}{\mathrm{mm^2}}\frac{\mathrm{m}}{\mathrm{s}} < (pv)_{limit} = 1.6\frac{\mathrm{W}}{\mathrm{mm^2}}$$

COMMENT The heating verification is not necessary because the journal is running slowly ($n < 900$ rpm). However, if the W factor is calculated, the bearing belongs to the lower class of *Table 44.5* with $W_{all} = 15{,}000 \div 20{,}000$ (N/mm)(rpm).

$$\textbf{44-9}:\ W = \frac{Fn}{l} = \frac{3000\ \mathrm{N} \times 130\ \mathrm{rpm}}{25\ \mathrm{mm}} = 15{,}600 \leq W_{all}$$

44.2.3 Rolling-contact bearings

The main elements of the **rolling bearings**[44.1] are an *outer ring* in a housing and an *inner ring* on a shaft seat; the rolling element (ball, cylindrical roller, needle roller, tapered roller, spherical roller, toroidal roller) is between these two rings and is usually kept at a given distance by a cage (*Figure 44.13-a*). The main dimensions of a bearing comprise the outside diameter D, the bore diameter d, the width B and the chamfer radius r to be accounted to design the mounting seats (*Figure 44.13-b*). The rolling bearings have lower values of friction and, in general, of wear with respect to sliding bearings. On the other hand, the roller bearings show limits on:

– the rotational speed because of rotating mass inertia;
– the loading capacity because of the highest pressure between the small contact area between rolling element and raceway (this is particularly true for ball bearing: roller bearing have a greater load-carrying capacity than ball bearing of equivalent size as they have line contact rather than point contact with their rings).

At the same loading, the higher radial bulk of rolling bearings is counterbalanced by the higher axial bulk of journal bearings.

44.1 - Journal bearings can support only radial and axial loads. On the contrary, angular contact ball bearings are designed to combine radial and axial loading.

Fig. 44.13-a - Self-aligning double-row barrel roller bearings.

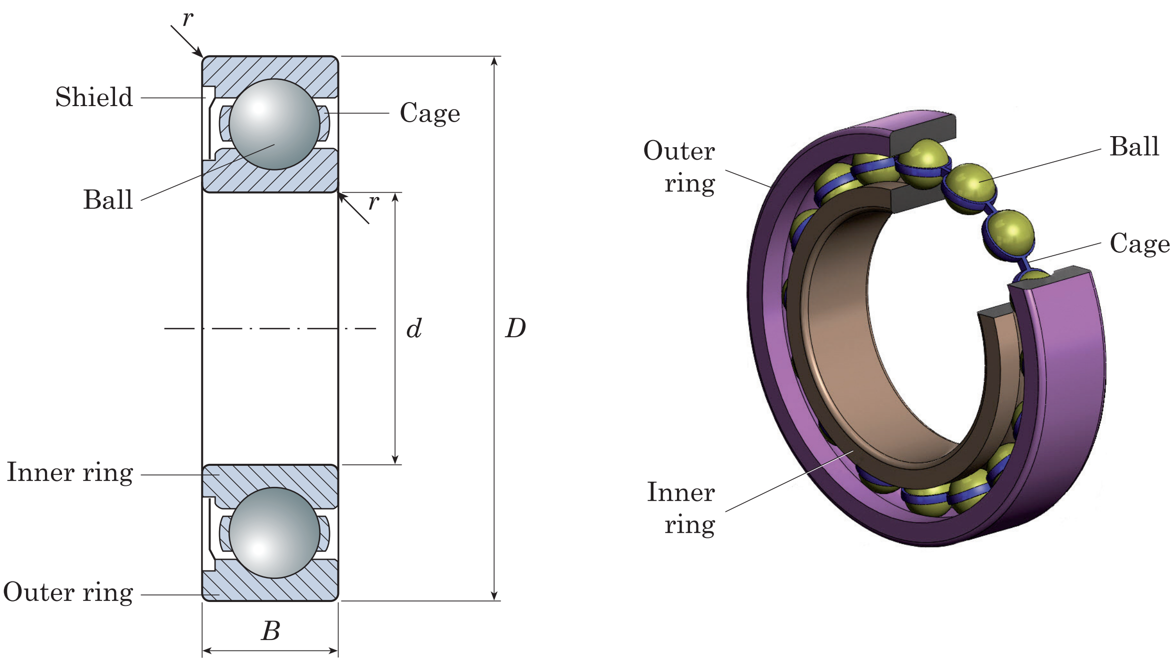


Fig. 44.13-b - Nomenclature of a ball bearing.

Shafts are generally supported by two bearings in the radial and axial directions (*Figure 44.14*). The side that fixes relative movement of the shaft and housing in the axial direction is the *fixed side bearing*, and the side that allows movement is the *floating side bearing*; the floating side bearing is needed to absorb mounting error and to avoid stress caused by expansion and contraction of the shaft due to temperature change. Angular contact cylindrical roller bearing are designed to combine radial and axial loading (*Figure 44.15*). Single direction or double direction thrust ball bearings are designed to accommodate axial loads only and must not be subjected to any radial load (*Figure 44.16*).

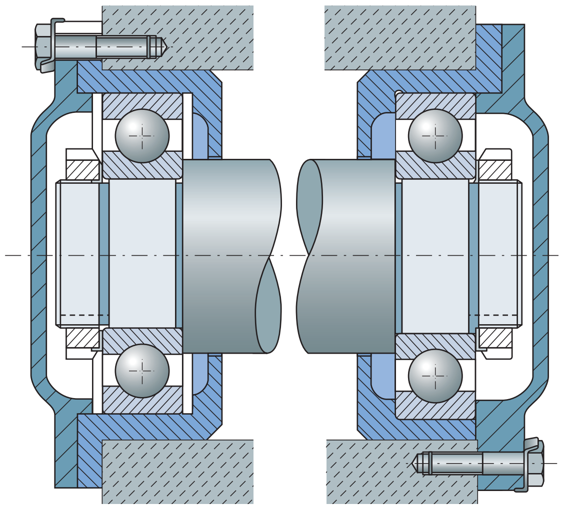


Fig. 44.14 - Mounting of radial rolling bearings: the outer ring (on the left) is not axially blocked. In a couple of rigid bearings mounted on a shaft, only one must be axially blocked, while the other one must be free to account eventual longitudinal expansion or contractions of the shaft.

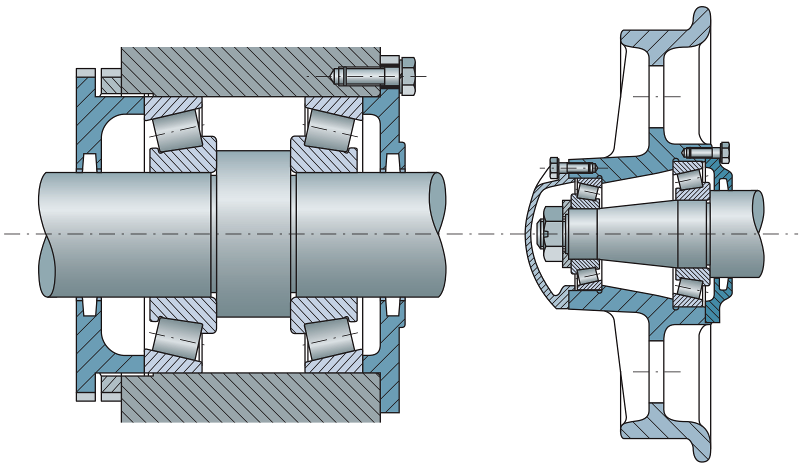


Fig. 44.15 - Angular contact cylindrical roller bearing are designed to combine radial and axial loading.

Right: back-to-back arrangement. Mounting two bearings back-to-back provides a relatively stiff bearing arrangement, which can also accommodate tilting moments. When arranged back-to-back, the load lines diverge towards the bearing axis.

Left: face-to-face arrangement. Mounting two bearings face-to-face is not as stiff as a back-to-back arrangement, but less sensitive to misalignment. When arranged face-to-face, the load lines converge towards the bearing axis.

The various types of rolling bearings can be found in catalogues of manufacturers, such as that of SKF. The individual life L of a rolling bearing is expressed as the number of revolutions or the number of operating hours at a given speed that the bearing is capable of enduring before the first sign of metal fatigue occurs on a raceway of the inner or outer

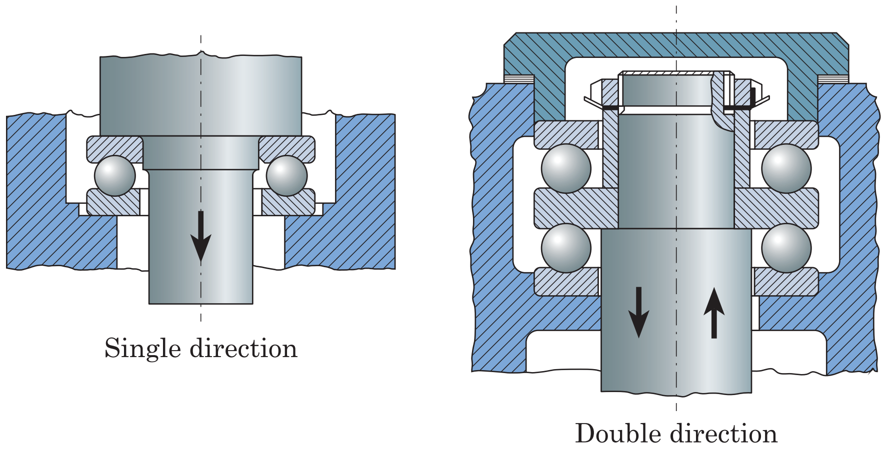


Fig. 44.16 - Single direction or double direction thrust ball bearings are designed to accommodate axial loads only and must not be subjected to any radial load. Single direction thrust ball bearings consist of a shaft washer, a housing washer and a ball and cage assembly; they can accommodate axial loads and locate a shaft axially, in one direction only. Double direction thrust ball bearings consist of one shaft washer, two housing washers and two ball and cage assemblies; they can accommodate axial loads and locate a shaft axially, in both directions.

ring or on a rolling element. A bearing is typically selected on the basis of its load rating relative to the applied loads and the requirements regarding bearing life and reliability (*Table 44.7*). The life of rolling bearings is determined by reading on manufacturer's catalogue the basic load ratings dynamic C and static C_0 as a function of principal dimensions as bearing bore and outside diameter, and length:

– the *static load rating* C_0 [N] is used in case of bearings having very low rotational speeds or normal speeds but subjected to heavy, short duration shock loads that may occur on rare occasions. It is the load under which the permanent deformation induced in the rolling element (or raceways) is equal to $1/10{,}000$ of its diameter;

– the *dynamic load rating* C [N] is the load under which the 90% of the sampled bearings exceeds a theoretical life of one million of revolutions before a failure appears. It is tabulated in the manufacturers' catalogue and is valid until 120 °C. At higher temperatures, the following corrections of dynamic load C are applied: 1.00 at 150 °C, 0.90 at 200 °C, 0.75 at 250 °C and 0.60 at 300 °C.

The fatigue life L of rolling bearings is based on a relationship between the dynamic load rating C and the slope of the fatigue curve for that type of bearing; the life L is expressed as million of cycles (revolutions) or operating hours (L_h). Being the bearing subjected both to radial and axial loads, the load is a combination of these two loads as a function of the bearing type; this load is called *equivalent dynamic bearing load* and indicated with P [N].

$$L = \left(\frac{C}{P}\right)^p \text{ [million of rev]} \qquad \text{or} \qquad L_h = \frac{10^6}{n \cdot 60}\left(\frac{C}{P}\right)^p \text{ [hours at a speed } n\text{, rev/min]} \qquad \textbf{44-14}$$

with $p = 3$ for ball bearings and 10/3 for roller bearings.

Table 44.7
Life of rolling bearings

Machinery	*Life* [Operating hours]
Household machines, agricultural machinery, instruments, technical equipment for medical use.	300 ÷ 3000
Machinery used for short periods or intermittently: electric hand tools, lifting tackle in workshops, construction equipment and machines.	3000 ÷ 8000
Machinery used for short periods or intermittently where high operational reliability is required: lifts (elevators), cranes for packaged goods or slings of drums, etc.	8000 ÷ 12,000
Machinery for use 8 hours a day, but not always fully utilized: gear drives for general purposes, electric motors for industrial use, rotary crushers.	10,000 ÷ 25,000
Machinery for use 8 hours a day and fully utilized: machine tools, woodworking machines, machines for the engineering industry, cranes for bulk materials, fans, conveyor belts, printing equipment, separators and centrifuges.	20,000 ÷ 30,000
Machinery for continuous 24 hour use: rolling mill gear units, medium-size electrical machinery, compressors, mine hoists, pumps, textile machinery.	40,000 ÷ 60,000
Hydraulic machinery, rotary furnaces, propulsion machinery for ocean-going vessels.	60,000 ÷ 100,000
Very high reliable operating machinery 24 hours a day: large electric machinery, power generation plant, mine pumps and fans, tunnel shaft bearings for ocean-going vessels.	> 100,000
Vehicles	*Life* [Million of kilometers]
On- and off-road vehicles	
Passenger cars	0.3
Trucks and buses	0.6
Railways vehicles	
Freight wagons, light rail and tramway vehicles	1.5
Main line passenger coaches	2
Local diesels	4
Main lines diesels	4 ÷ 6
Main line diesel and electric locomotives	5 ÷ 8

The equivalent dynamic bearing load P [N] is the hypothetical load, constant in magnitude and direction, acting radially on a radial bearing or axially and at the center line on a thrust bearing which, if applied, would have the same influence on bearing life as the actual loads to which the bearing is subjected. Radial bearings are often subjected to simultaneously acting radial and axial loads. If the resultant load is constant in magnitude and direction, the equivalent dynamic bearing load P can be obtained from the general equation:

$$P = XF_r + YF_a \qquad \textbf{44-15}$$

where:

P = equivalent dynamic bearing load [N];

F_r = actual radial bearing load [N];

F_a = actual axial bearing load [N];

X = radial load factor for the bearing;

Y = axial load factor for the bearing;

with factors X and Y depending on F_a/F_r ratio (see manufacturers' catalogues).

44.3 Critical speeds

If a disk mounted upon a shaft rotates about it, the center of gravity of the disk must be at the center of the shaft, if a perfect running balance has to be obtained. But, if the center of gravity of the disk is slightly moved away from the center of the rotating shaft, the centrifugal force generated by the heavier side will be greater than that generated by the lighter side geometrically opposed to it and the shaft will deflect toward the heavier side, causing the center of disk to rotate in a small circle. The same occurs for the axis of a shaft that, because of an insufficient lathe work, does not form a perfect straight line. When this shaft is turning, eccentricity causes a centrifugal force deflection, which is resisted by (*Paragraph 20.3*) the shaft's flexural rigidity EI, where E is the modulus of elasticity and I is the moment of inertia of the shaft. At certain speeds, the deflections, that at the very beginning could be very small, increase without upper bound: they are called the *critical speeds* of the system with deformations which, in the absence of damping, could tend to an infinite amplitude.

The rotating shaft, because of its own mass, has a *critical speed* that excites its *natural frequency* (*Paragraph 8.6*). When geometry is simple, as in a shaft of uniform diameter, simply supported, the critical speed $\omega_{\mathrm{c,alb}}$ as a function of shaft mass m_{shaft}, support distance l and flexural stiffness EI is given by:

$$\omega_{\mathrm{c,shaft}} = \pi^2 \sqrt{\frac{EI}{m_{\mathrm{shaft}} l^3}} \qquad \textbf{44-16}$$

The ensemble of attachments to a shaft likewise has a critical speed that is much lower than the shaft's intrinsic critical speed. By considering a shaft with masses m_1, m_2, m_3, …, m_n, it is possible to identify a critical speed ω_{c1} which considers only the mass m_1 acting alone, likewise a critical speed ω_{c2} considering m_2 acting alone, and so on. Each of these is expressed by $\sqrt{g/|y|}$ with $g = 9.81$ m/s^2 acceleration due to gravity and $|y|$ [m] absolute value of deflection related to the given mass. As an example, the critical speed of the mass m_1 is $\omega_{c1} = \sqrt{g/|y_1|}$ where $|y_1|$ is the shaft deflection due to m_1 acting alone, the critical speed of the mass m_n is $\omega_{cn} = \sqrt{g/|y_n|}$ where $|y_n|$ is the shaft deflection due to m_n acting alone. The critical velocity $\omega_{\mathrm{c,shaft}}$ given by the equation **44-16** can be also considered if the own mass of the shaft has to be taken into account. With *Dunkerley's equation* the first, or fundamental, critical speed, ω_c of the mass ensemble can be approximated by:

$$\frac{1}{\omega_c^2} = \frac{1}{\omega_{c1}^2} + \frac{1}{\omega_{c2}^2} + \frac{1}{\omega_{c3}^2} + \ldots + \frac{1}{\omega_{cn}^2} \quad \text{with} \quad \omega_{cn} = \sqrt{\frac{g}{|y_n|}} \qquad \textbf{44-17}$$

By ignoring the higher mode term(s), the first critical speed estimate is *lower* than actually is the case. Then, in applying equation **44-17**, the angular velocity of the shaft $\omega = \frac{2\pi n}{60}$ has to be the 80% of the first critical speed: $\omega < 0.8 \cdot \omega_c$ (or $n < 0.8 \cdot n_c$).

The absolute value of the rotating shaft deflection y should be compared to the extension y (here y substitutes the coordinate x quoted in *Paragraph 8.4*) of the mass m of a spring having a stiffness $k = F/y$ under the weight $F = mg$ (**1-10′**). By taking into account the period (**8-5**) $T = (2\pi)/\omega$ of the simple harmonic motion and the spring period (**8-7**) $T = 2\pi\sqrt{m/k}$, the formula **44-17** of the critical velocity of the single mass is obtained:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad \Rightarrow \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{k\frac{1}{m}} = \sqrt{\frac{F}{y}\frac{1}{\frac{F}{g}}} = \sqrt{\frac{g}{y}} \qquad \textbf{44-18}$$

Example 44.3 Critical speed of a shaft

Consider a simply supported steel shaft as depicted in *Figure 44.17*, with $d = 50$ mm diameter and $l = 1$ m span between bearings and designed to support the mass m_1 ($P_1 = 250$ N) and the overhanging mass m_2 ($P_2 = 450$ N). By neglecting the own mass of the shaft, find the first critical speed and, by knowing the rotational speed $n = 600$ rpm, establish if the rotating system is safe.

SOLUTION

Table VII: $I = \dfrac{\pi d^4}{64} = \dfrac{\pi \times (0.05\ \text{m})^4}{64} = 3.07 \times 10^{-7}\ \text{m}^4$

44-6, B.2-10: $|y_1| = \dfrac{P_1 a^2 b^2}{3EIl} = \dfrac{250\ \text{N} \times (0.4\ \text{m})^2 \times (0.6\ \text{m})^2}{3 \times 206 \times 10^9\ \text{Pa} \times 3.07 \times 10^{-7}\ \text{m}^4 \times 1\ \text{m}} = 7.6 \times 10^{-5}\ \text{m}$

44-6, B.2-10: $|y_2| = \dfrac{P_2 c^2}{3EI}(l + c) = \dfrac{450\ \text{N} \times (0.5\ \text{m})^2}{3 \times 206 \times 10^9\ \text{Pa} \times 3.07 \times 10^{-7}\ \text{m}^4} \times (1\ \text{m} + 0.5\ \text{m}) = 8.89 \times 10^{-4}\ \text{m}$

44-17: $\omega_{c1} = \sqrt{\dfrac{g}{|y_1|}} = \sqrt{\dfrac{9.81\ \text{m/s}^2}{7.6 \times 10^{-5}\ \text{m}}} = 359\ \text{rad/s} \quad \omega_{c2} = \sqrt{\dfrac{g}{|y_2|}} = \sqrt{\dfrac{9.81\ \text{m/s}^2}{8.89 \times 10^{-4}\ \text{m}}} = 105\ \text{rad/s}$

44-17: $\frac{1}{\omega_c^2} = \frac{1}{\omega_{c1}^2} + \frac{1}{\omega_{c2}^2} = \frac{1}{(359\ \text{rad/s})^2} + \frac{1}{(105\ \text{rad/s})^2} = 9.85 \times 10^{-5}\left(\frac{\text{s}}{\text{rad}}\right)^2 \quad \Rightarrow$

$$\Rightarrow \quad \omega_c = \sqrt{\frac{1}{9.85 \times 10^{-5}\left(\text{s/rad}\right)^2}} = 100.8\ \text{rad/s} \qquad ◀$$

6-11: $\omega = \frac{2\pi n}{60} = \frac{2 \times \pi \times 600\ \text{giri/min}}{60} = 62.8\ \text{rad/s}$

$\omega < 0.8 \cdot \omega_c \quad \Rightarrow \quad 62.8\ \text{rad/s} < 0.8 \times 100.8\ \text{rad/s} = 80.6\ \text{rad/s}$ << the system is safe >> ◀

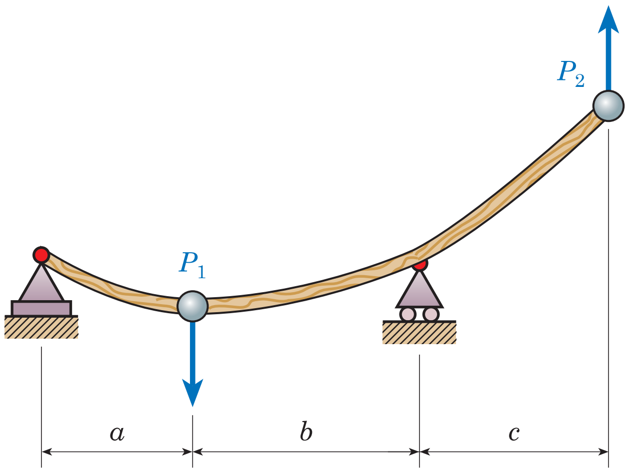


Fig. 44.17 - Shaft of *Example 44.3* with $a = 0.4$ m, $b = 0.6$ m and $c = 0.5$ m.